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A COTTONPICKIN' COTTON GINNING PROBLEM

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13. ABSTRACT

In the Mesilla Valley of New Mexico there are 150 cotton farmers and 20 cotton gins. The weekly cotton picking and the weekly gin capacities are known, as well as the transportation and ginning costs. The objective is to determine not only the shipping schedule for shipping cotton to gins but also which gins should be opened. This paper shows how network analysis can be used not only to visualize such a model but also to develop a computationally feasible algorithm.

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A COTTONPICKIN' COTTON GINNING PROBLEM

by

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May 1974

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INTRODUCTION

This paper addresses the real world problem faced by 150 cotton farmers and 20 cotton gin operators in the Mesilla Valley of New Mexico and illustrates the importance of considering alternative formulations of large scale mathematical programming problems in order to reduce their size and complexity. The weekly cotton production of each farmer for the approximately 20 weeks of cotton picking are known. Also known are the weekly gin capacities. Since the transportation costs are available, it may seem to be an ordinary transportation problem. However, both the weekly and the seasonal ginning cost functions must also be considered and these are nonlinear with a fixed charge on the seasonal function. Furthermore, it is possible to store cotton from one week to the next, but at a price. Thus, the problem of scheduling the shipment of cotton from farms to gins becomes a large scale nonlinear programming problem.

In this paper we first illustrate how this problem can be formulated as a large scale mixed integer transportation problem with extra linear constraints, and outline a possible solution procedure using decomposition. Unfortunately, the transportation part of this formulation yields a problem

with 4,201 nodes and 2,460,000 arcs. While this size transportation problem is within the solution limits of state-of-the-art transportation computer codes [4], the extra linear constraints and the 0-1 integer variables imply that the transportation problem will have to be solved several times and the repetitive solution of such large scale transportation problems may not be computationally feasible.

By fully exploiting the topological characteristics of the problem, we show that this problem can also be formulated as a fixed charge minimum cost flow network consisting of 3,441 nodes, 61,640 arcs and twenty 0-1 variables associated with the operation or nonoperations of the gins. Such a problem can easily be solved by branch and bound procedures [2] if codes are used which exploit the structure of the minimum cost flow subproblems. For example, the fastest known minimum cost flow network code [4], which can handle a problem of this size, solves minimum cost flow network problems with 3,000 nodes and 60,000 arcs in less than four minutes on a CDC 6600.

For completeness, the paper concludes with a description of a branch and bound algorithm and the solution of a 5 farm, 4 gin example. The primary purpose for the inclusion of this material is to illustrate the entire Operations Research modeling and solution process.

THE PROBLEM

This cotton gin problem is designed to minimize the costs involved in processing cotton. The costs involved are storage costs, shipping costs, and gin-processing costs. The problem involves 150 cotton farms and 20 cotton gins in the Mesilla Valley of New Mexico. Each of the farms has cotton ready for ginning each week during a twenty-week picking period.

This cotton may be shipped immediately to a gin or may be stored on the farm and shipped at a later date. Because cotton is a seasonal crop, it is not feasible to keep the gins operating all year round. Thus, the gins are available for ginning cotton only during the twenty-week production period plus an additional ten or so weeks. During the five month period when no cotton is available for ginning, all gins are shut down. Before the picking of cotton actually begins, planting information leads to quite accurate forecasts of the amount of cotton that will be available for shipping from each of the farms during each week. Further, not all cotton picked in any given week need be ginned in that week. For example, during a particularly productive week, it may be preferable to store cotton and have it ginned the following week.

One of the decisions which has to be made each year is how many of the gins should be put into operation. In recent years the production of cotton has been going down and thus the total production for a cotton picking season is now considerably less than the seasonal capacity of the twenty gins. Thus, it may be feasible to operate only some of the gins. This decision is dependent on the cost of running each gin and its efficiency. For each gin there is a seasonal cost function that is a convex, piecewise linear function with a fixed charge, an example of which is illustrated by Figure 1. The fixed charge represents one time charge for activating a gin that has been closed down for five months, and includes such things as utility hook-up charges, clearing, and hiring of personnel. The variable costs in this seasonal gin cost function are due largely to the cost of electric power. The initial seasonal variable costs are lower than the regular costs because most of the initial costs are

absorbed by the hook-up charges.

Further, the total weekly capacity of each gin is divided into the regular capacity and an additional capacity available if an overtime shift is utilized. Thus, there are two levels of variable direct labor costs associated with each gin--one for the regular shift and another for overtime. If the capacity of the regular shift is exceeded, all of the additional cotton must be processed at this more expensive overtime rate. However, prudent use of this overtime may be profitable if it avoids the necessity of activating an additional gin.

The solution to the problem of minimizing shipping, storing, and ginning costs yields information necessary for implementation of this least-cost program. In particular, the solution designates which gins should be activated for processing cotton, how much cotton should be stored on each farm in each week, how much cotton each farm should ship each week and to which gins, and how much cotton should be ginned at overtime rates at each of the gins being used.

There are nine main factors which affect the optimal solution to this problem:

(1) The cost of shipping cotton from each farm to each gin.

(2) The start-up cost for each gin.

(3) The capacity of each gin's regular shift and its overtime shift.

(4) The variable weekly costs of ginning at regular rates.

(5) The variable costs of ginning at overtime rates.

(6) Each farm's holding costs for storing cotton.

(7) The variable initial utility rates.

(8) The variable regular utility rates.

(9) The transition point in seasonal gin production between initial and regular rates.

EXAMPLE DATA

In order to study the structure of this problem better, a small example of five farms which produce cotton available for shipping each week for three weeks will be used. There are four gins which may operate for these three weeks plus an additional three weeks. Each gin has two levels of weekly costs for ginning. The first level is applicable to all cotton ginned during the regular shift, while the second level applies to all cotton ginned during the overtime shift. Also each gin has a seasonal start-up cost and two levels of seasonal costs. The seasonal costs, the regular shifts' capacities with their associated costs, and the overtime shifts' capacities with the associated overtime costs are given in Table I. The production level for each of the five farms during the three weeks of cotton picking, the shipping costs from each farm to each gin, and the holding cost for storing cotton on the farms is also included in Table I. The next sections develop different formulations for this problem.

TABLE 7

PRODUCTION, COSTS, AND CAPACITIES

SHIPPING COSTS

	Gin			
Farm	1	2	3	4
1	6	2	4	7
2	3	5	8	4
3	2	6	2	9
4	5	3	7	6
5	4	5	9	4

PRODUCTION

	Farm					
Week	1	2	3	4	5	Total
1	15	40	30	15	15	115
2	35	75	45	50	50	255
3	20	60	20	35	20	155

Total Production 525

GIN CAPACITIES

Gin	Regular Shift	Overtime Shift	Total	6-Weeks Total
1	20	10	30	180
2	15	10	25	150
3	20	10	30	180
4	50	20	70	420

Total Capacity 930

GIN COSTS

Weekly Costs

Gin	1	2	3	4
Regular per-bale Cost	1	2	2	1
Overtime per-bale Cost	3	7	5	2

Seasonal Costs

Start-up Cost	300	200	500	650
Initial per-bale Cost	1	6	2	1
Regular per-bale Cost	6	10	7	5
Transition Point between initial and regular rate in bales	40	20	30	70

Holding Cost

Cost per-bale per-week = 1

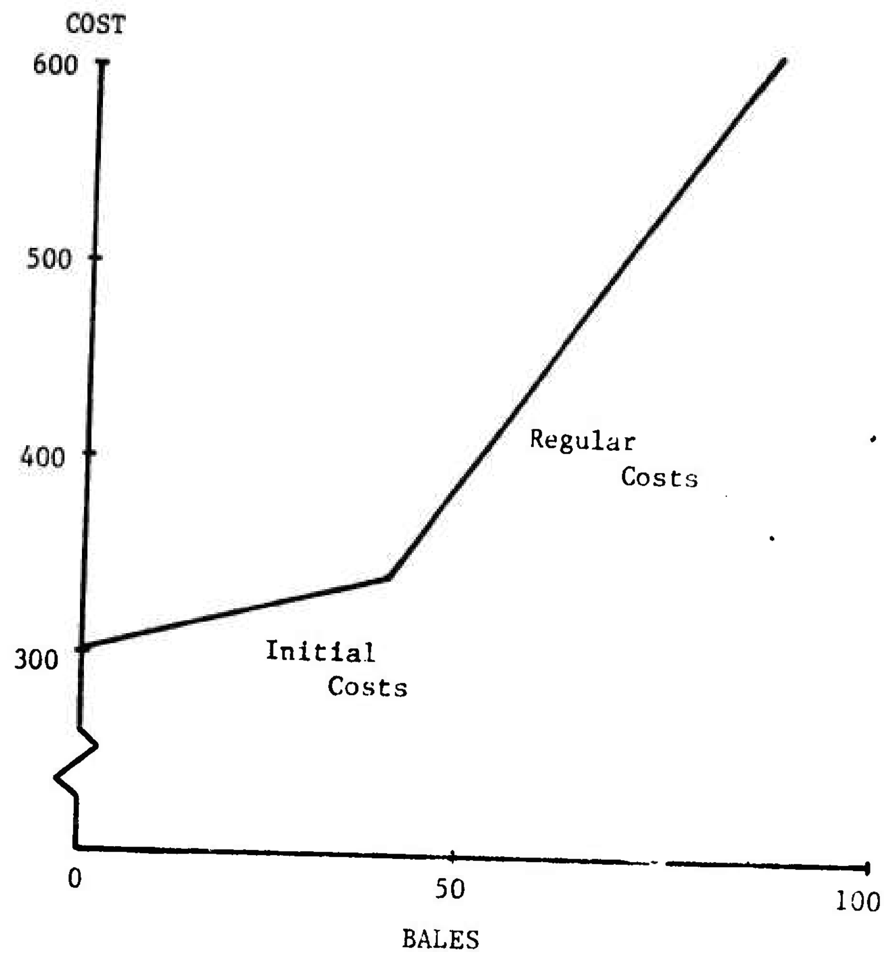

Figure 1

SEASONAL GINNING COSTS FOR GIN 1

FORMULATION ONE

In order to solve the problem of minimizing the total costs associated with storing cotton, transporting it to gins, starting the gins, and ginning the cotton, we first considered decomposing the problem into two subproblems. One of the subproblems is a weekly problem in which the costs of shipping cotton to gins, storing cotton at farms, and ginning labor costs are minimized. In the second subproblem the total seasonal ginning costs are minimized.

Consider first the weekly subproblem. A farm may ship the cotton picked during its first week to any of the four gins, or it may store the cotton on the farm at a cost of $1 per-bale per-week and ship it to a gin for processing during any of the next five weeks. Similarly, during the other cotton picking weeks, a farm may ship the production to a gin or store it and ship it during any of the remaining weeks. Thus, a transportation tableau can be constructed for which there is an origin node for each farm for each week of production, and an additional origin node designating excess gin capacity for the six weeks of processing. Since there are five farms and three production weeks, this yields sixteen origin nodes. Each farm may ship to any of the four gins each week. For each processing week two destination nodes for each gin can be created. One of these two nodes has a demand representing the weekly capacity of the gin's regular shift; the second node has a demand representing the weekly capacity of the overtime shift. Since there are four gins and each may process for six weeks, this yields forty-eight destination nodes. Each farm may ship the cotton picked during a given week to any gin during the week it is picked or during any subsequent week. Thus, there are six

hundred variables representing shipments to gins. There are an additional 48 slack variables representing the excess of gin capacity over farm production. This totals 648 variables and 64 constraint equations for the transportation problem.

Consider the costs for the cells in the transportation tableau of Table II. Each cost consists of three component costs (inadmissible cells have no costs indicated):

(1) The cost of shipping the cotton from the farm to the gin.

(2) The holding cost which is applied only if the cotton is stored rather than being shipped in the same week it is produced.

(3) The labor cost of the cotton gin to which the cotton is shipped.

In the seasonal subproblem there is for each gin a 0-1 variable and a constraint equation representing total seasonal gin production. If a gin is activated, then its 0-1 variable will be one, and this variable will incur a cost equal to the start-up cost of the gin; if the gin is not activated, the variable will be zero. Probably the most efficient way of finding these variables is by implicit enumeration. Also, if a gin is activated, it will have a corresponding constraint equation. The associated variables of this constraint equation will have cost values in the objective function that correspond to the variable seasonal ginning costs. This means that each set of 0-1 values defines a new set of seasonal constraint equations and a corresponding objective function, which together with the transportation subproblem can be solved by the usual decomposition procedures.

Generally, decomposition requires the solving of each of the subproblems over and over again, especially since the convergence often is slow. In

Table II Transportation Tableau for Formulation One

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	405
7	9	4	9	6	9	8	9	8	10	5	10	7	10	9	10	9	11	6	11	8	11	10	11	10	12	7	12	9	12	11	12	11	13	8	13	10	13	12	13	12	14	9	14	11	14	13	14	15
4	6	7	12	10	13	5	6	5	7	8	12	11	14	6	7	6	8	9	13	12	15	7	8	7	9	10	14	13	16	8	9	8	10	11	15	14	17	9	10	9	11	12	16	15	18	10	11	40
3	5	8	13	4	7	10	11	4	6	9	14	5	8	11	12	5	7	10	15	6	9	12	13	6	8	11	16	8	10	13	14	7	9	12	17	8	11	14	15	8	10	13	18	9	12	15	16	30
6	8	5	10	9	12	7	8	7	9	6	11	10	13	8	9	8	10	7	12	11	14	9	10	9	11	8	13	12	15	10	11	10	12	9	14	13	16	11	12	11	13	10	15	14	17	12	13	15
5	7	7	12	11	14	5	6	6	8	8	13	12	15	6	7	7	9	9	14	13	16	7	8	8	10	10	15	14	17	8	9	9	11	11	16	15	18	9	10	10	12	12	17	16	19	10	11	15
								7	9	4	9	6	9	8	9	8	10	5	10	7	10	9	10	9	11	6	11	8	11	10	11	10	12	7	12	9	12	11	12	11	13	8	13	10	13	12	13	35
								4	6	7	12	10	13	5	6	5	7	8	12	11	14	6	7	6	8	9	13	12	15	7	8	7	9	10	14	13	16	8	9	8	10	11	15	14	17	9	10	75
								3	5	8	13	4	7	10	11	4	6	9	14	5	8	11	12	5	7	10	15	6	9	12	13	6	8	11	16	8	10	13	14	7	9	12	17	8	11	14	15	45
								6	8	5	10	9	12	7	8	7	9	6	11	10	13	8	9	8	10	7	12	11	14	9	10	9	11	8	13	12	15	10	11	10	12	9	14	13	16	11	12	50
								5	7	17	12	11	14	5	6	6	8	8	13	12	15	6	7	7	9	9	14	13	16	7	8	8	10	10	15	14	17	8	9	9	11	11	16	15	18	9	10	50
																7	9	4	9	6	9	8	9	8	10	5	10	7	10	9	10	9	11	6	11	8	11	10	11	10	12	7	12	9	12	11	12	20
																4	6	7	12	10	13	5	6	5	7	8	12	11	14	6	7	6	8	9	13	12	15	7	8	7	9	10	14	13	16	8	9	60
																3	5	8	13	4	7	10	11	4	6	9	14	5	8	11	12	5	7	10	15	6	9	12	13	6	8	11	16	8	10	13	14	20
																6	8	5	10	9	12	7	8	7	9	6	11	10	13	8	9	8	10	7	12	11	14	9	10	9	11	8	13	12	15	10	11	35
																5	7	7	12	11	14	5	6	6	8	8	13	12	15	6	7	7	9	9	14	13	16	7	8	8	10	10	15	14	17	8	9	20
20	10	15	10	20	10	50	20	20	10	15	10	20	10	50	20	20	10	15	10	20	10	50	20	20	10	15	10	20	10	50	20	20	10	15	10	20	10	50	20	20	10	15	10	20	10	50	20	

addition, with the implicit enumeration, the number of these decomposition problems to be solved in itself could be very large. Since the actual problem of 150 farms and 20 gins would have transportation problems with up to 4,201 constraint equations and 2,460,600 variables, the computational effort appeared to be fantastically large. Therefore, alternate formulations seemed to be required.

FORMULATION TWO

This problem may be reformulated so as to reduce the number of variables and constraints. First, we set up a minimum cost flow network problem which ignores the fixed charges and includes only variable costs, including the seasonal variable costs. This is given by Figure 2. The enclosed area, which considers only the arcs from farm one to gin one, is shown more clearly in Figure 3.

This problem will have an origin node representing each farm for each of the shipping weeks. Level iAj nodes represent farm i in week j of Figures 2 and 3. The supply of each of these nodes is the amount picked at the farm during this week. The amount stored from one week to the next at each farm is represented by arcs which ship between each farm's origin nodes representing subsequent weeks. In the small example of five farms there are a total of thirty origin nodes for the six shipping weeks. An additional twenty-four nodes as represented by iBj in Figures 2 and 3 arise from the six weeks of availability for ginning for each of the four gins. All cotton processed in each gin during the six weeks is then channeled through a single node, called the weekly master node for the gin. Level iC node corresponds to the weekly master node for gin i. This structure eliminates the need for creating two arcs from each farm node to

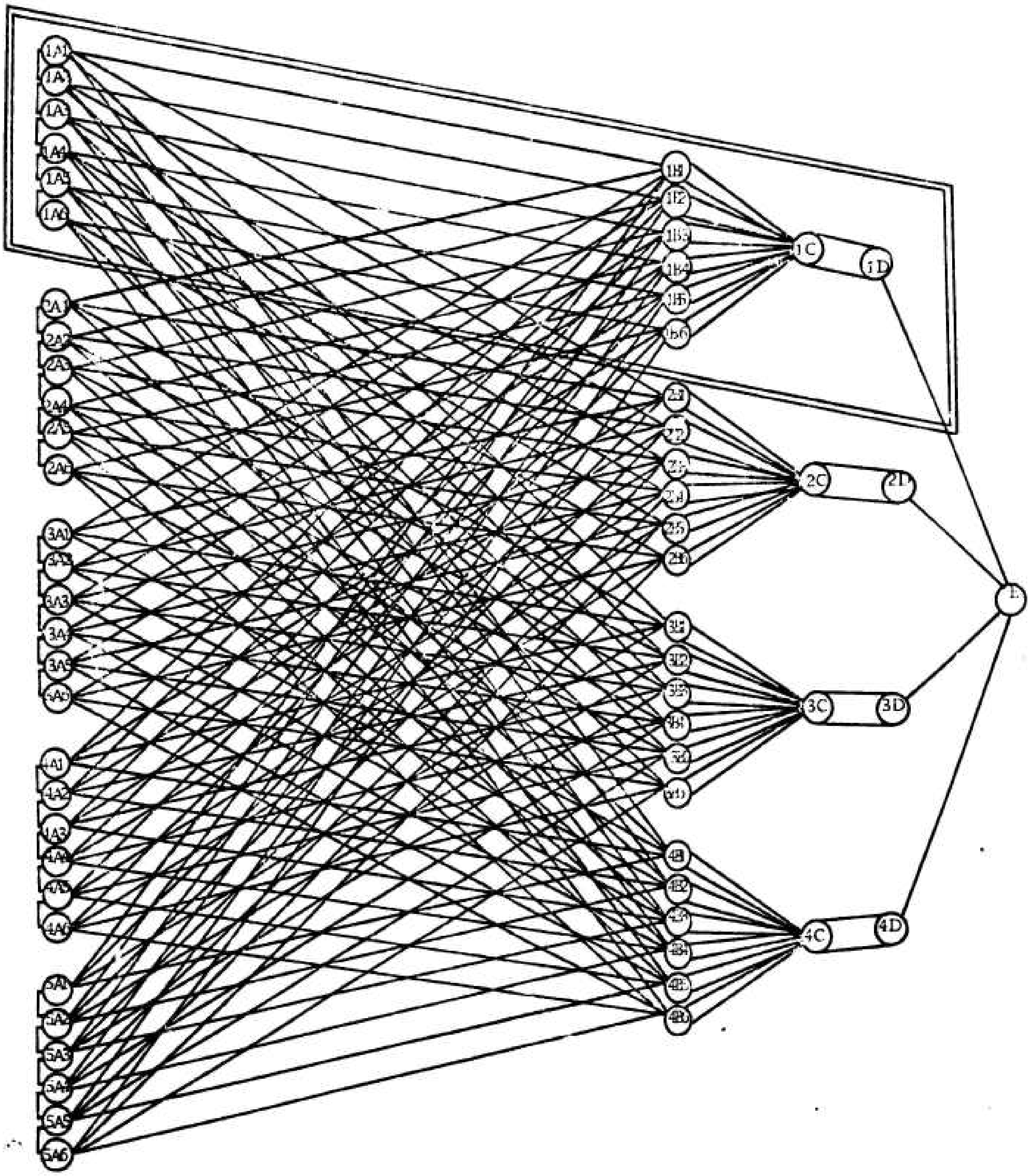

Figure 2

NETWORK DIAGRAM FOR FORMULATION TWO

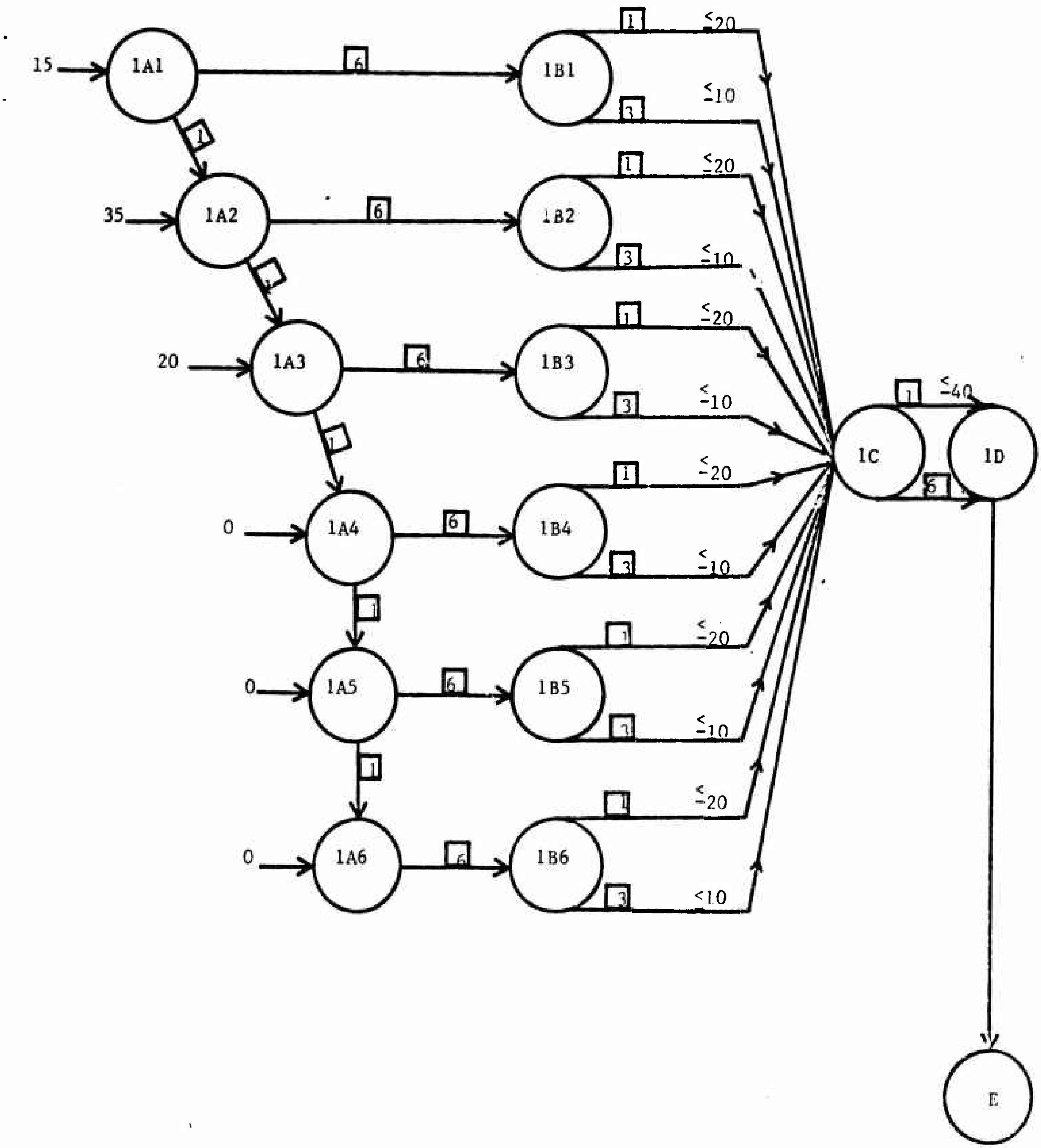

Figure 3

NETWORK DIAGRAM FOR FARM 1 AND GIN 1 OF FORMULATION TWO

accomodate the two weekly ginning costs, but adds four nodes representing the four gins. Another four nodes are also added to accomodate the variable seasonal costs. These nodes are represented by the D level nodes in Figures 2 and 3. Finally, all flow is channeled through a single node, node E, which acts as a sink for the entire production. By setting its demand equal to the total supply, the need for slack arcs is eliminated. This formulation of the problem contains a total of sixty-three nodes.

For each week the cotton picked on a farm for that week may be shipped to any one of the four gins or may be stored at the farm. Shipping to the gins is represented by four arcs (one to each gin), while storage at the farm is accomplished by shipping the cotton to the source node representing the subsequent shipping week of that farm. The effect of storing the cotton is to increase the amount available for next week's shipping. For each of the five farms, each week's production may be shipped to any of four gins or to a subsequent week's shipping node at the same farm. Since there are five farms and six weeks, and since the cotton for the last week cannot be stored, there are 145 shipping and storage arcs.

Each gin has a node for each week of ginning. For a given single week, the amount shipped to the weekly master node of a gin is limited to the weekly capacity of the gin. In order to represent the two levels of labor costs, there are two arcs representing the weekly labor costs at each gin. One of these arcs has an associated cost equal to the labor cost of the regular shift, and is capacitated by the amount of cotton which can be ginned during the regular shift. The second arc has an associated cost equal to that of the overtime shift and is capacitated by the amount of cotton which can be ginned in overtime. For each week of ginning at each

of the four gins there is a set of the two arcs. Since the two arcs ship between the same two nodes, the arc with the higher cost will not be used until the arc with the lower cost has reached its capacity; this insures that no overtime is used until the capacity of the regular shift has been reached. Since there are six weeks of processing and there are four gins, these twenty-four sets of arcs add forty-eight additional capacitated arcs to the network.

In addition, from each gin's weekly master node there are two more arcs leading to the gin's seasonal master node. These two arcs represent the initial and regular seasonal ginning costs. The capacity of the arc associated with initial costs is the number of bales at the transition point between initial and regular costs. These add eight more arcs, but at the same time eliminates the need for decomposing the problem into two subproblems.

Finally, the single arc from each gin's seasonal master node to the sink node adds four more arcs. This gives a total of 205 arcs for the network formulation. This reformulation not only has reduced the number of variables from 648 to 205, but also it has included the seasonal ginning costs. It should be noted, however, that 52 of these arcs are capacitated. This reduction in number of arcs has come from splitting each of the costs in the transportation problem into its three components: the shipping component, the storage component, and the labor component. Various combinations of these three component costs can duplicate all of the costs in the transportation tableau.

The number of nodes in this reformulation has decreased from 69 to 63 in spite of the addition of the seasonal ginning costs. This net reduction

in the number of nodes results from an increase due to having an origin node for each farm for each shipping week instead of merely for each production week and from a decrease due to the need for only one node for each week for each gin.

The corresponding reduction on the 150 farm problem results in the number of variables being reduced from 2,460,600 to 95,610. The number of nodes will increase from 4,201 to 5,141. Such a minimum cost flow network problem is well within the solution capability of state-of-the-art network codes; e.g., the code in [4] can solve problems of this size in less than 6 minutes on the CDC 6600.

FURTHER REDUCTIONS

Since it has been assumed that the shipping costs are constant over the season, and since the holding costs are the same on all farms, this problem may be reduced even more. It is not important to know which farm actually stores the cotton. Just knowing how much cotton each farm sends to each gin and how much of the cotton to be ginned at a given gin must be stored is sufficient information.

Since no farm picks cotton in weeks four, five, and six, the only reason these nodes exist is to show how much stored cotton is ginned by each gin in weeks four, five, and six. Since the assumption has been made that weekly storage costs are the same at each farm, one may just as well consider that all cotton to be ginned in weeks three, four, five, and six is shipped in the third week and stored at the gin. While this is not actually the case, this assumption will not change the optimal solution. Although setting up arcs which enable one to simulate storage at each gin during weeks three, four, and five adds three arcs at each of the four gins,

it enables one to eliminate these storage arcs in weeks three, four, and five at each of the five farms. As long as there are less gins than there are farms, this change results in a net reduction in the number of arcs. Since there actually is no cotton picked at the farms during weeks four, five, and six, the only cotton available for shipping is that stored from previous weeks. With the cotton now being stored at the gins instead of the farms in weeks three, four, and five, there is no cotton available for shipping from the farms during these last three weeks of ginning and the nodes representing weeks four, five, and six may be removed from the problem without changing the solution. This eliminates 15 nodes and 75 arcs while adding twelve new arcs. This reduction is illustrated in Figure 4. In the 150 farm problem the corresponding reduction eliminates 1,500 nodes and 31,500 arcs while adding 200 arcs, so that this problem now has 3,641 nodes and 64,310 arcs.

Since there are more farms than gins, an additional reduction may be made by storing cotton at the gins for all weeks instead of storing at the farms. In the five farm example there are presently ten arcs used to denote storage of cotton for weeks one and two. By moving this storage to the gins this number is reduced to eight, a reduction of two, and therefore this problem now has 140 variables and 48 nodes. The reduction is illustrated in Figure 5. The corresponding reduction in the 150 farm problem results in elimination of 2,470 arcs, leaving 61,340 variables and 3,641 nodes.

A final reduction may be made because the nodes representing the additional weeks of ginning for each gin are dummy nodes. In weeks four, five, and six, all flow passing a node in a gin is due only to cotton

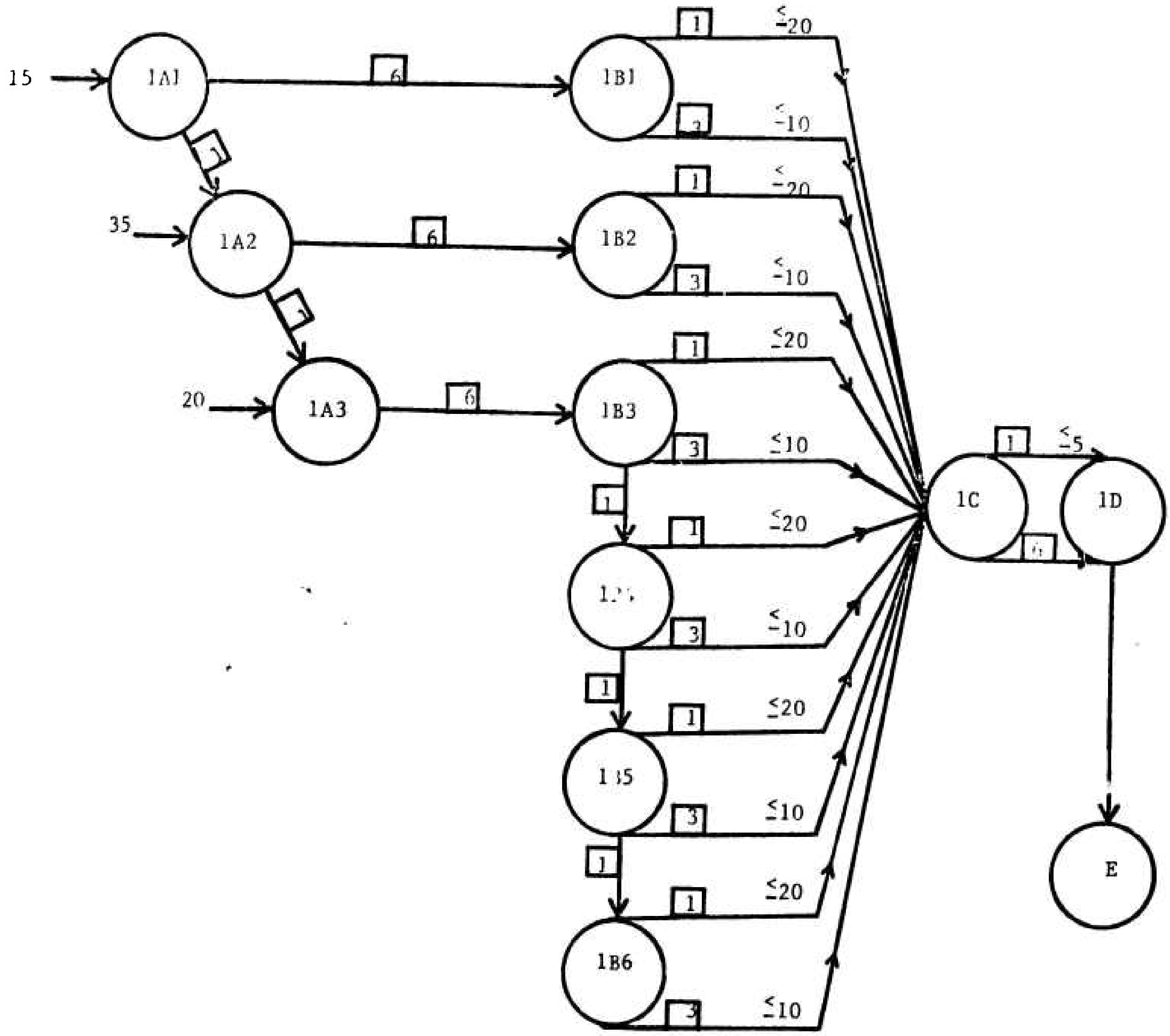

Figure 4

NETWORK AFTER REMOVAL OF THREE ORIGIN NODES

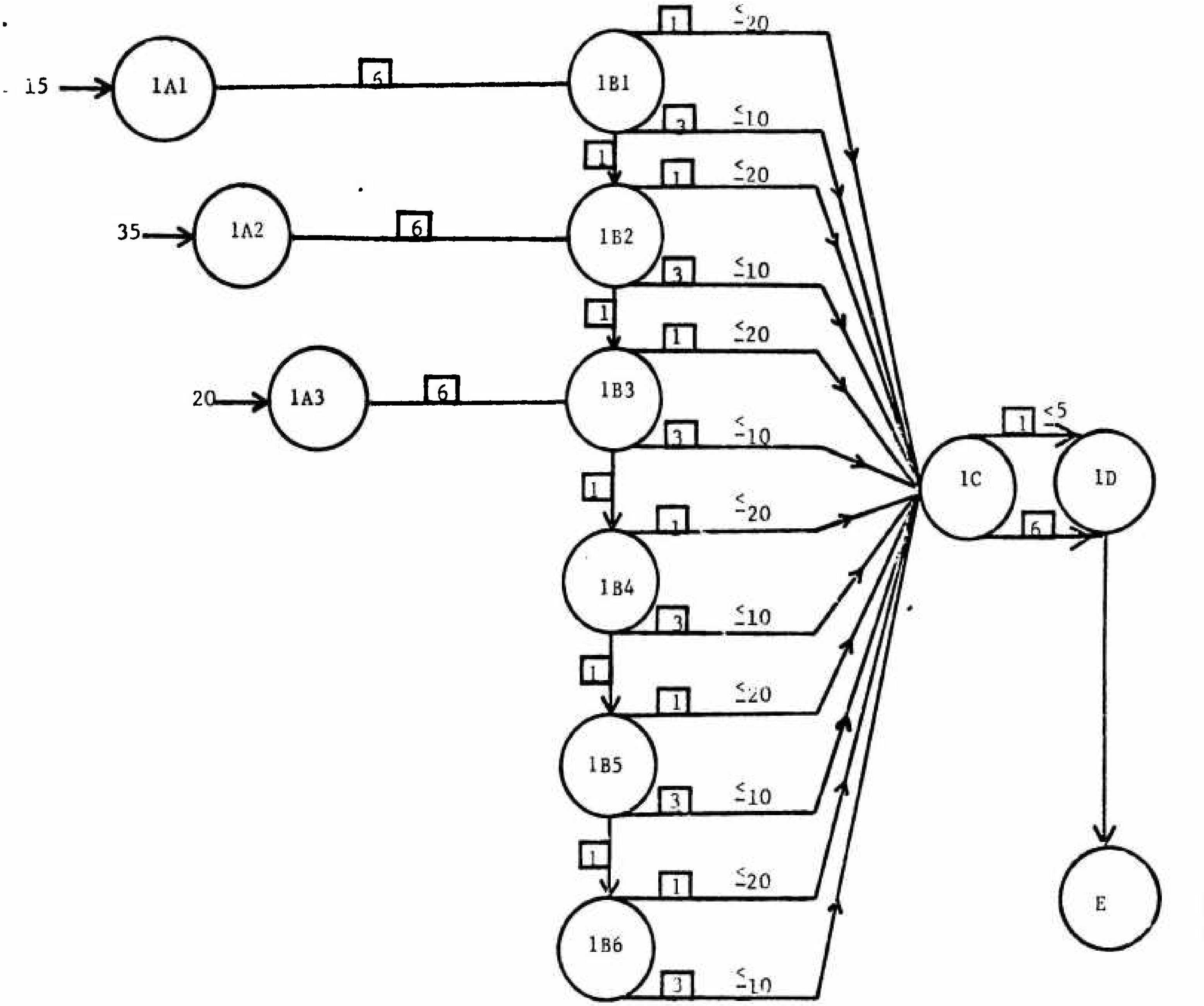

Figure 5

NETWORK WITH ALL COTTON STORED AT GINS

stored from the previous week. During these weeks there is no currently picked cotton to be ginned. The nodes representing weeks four, five, and six may be eliminated. The arcs from each of these nodes to the weekly master node for the gin can be tied directly to the node representing the third week of ginning at the gin. Each of these arcs incurs an additional cost equal to the storage costs of the now-eliminated storage arcs. One set of arcs represents cotton ginned in the third week. A second set of arcs from this same node represents cotton stored for one week and then ginned in the fourth week. Similarly, the other two sets of arcs will represent cotton stored for two weeks and three weeks before being ginned. Obviously, the optimal solution will always gin as much cotton as possible rather than storing it for an additional week and then ginning it at the same gin. The only exception might be if the overtime processing cost for one week is more expensive than storing the cotton for a week and then processing it at regular rates. This reduction removes twelve nodes and twelve arcs from this problem, leaving 128 variables and 36 nodes as shown in Figure 6. The 150 farm problem is reduced by two hundred arcs and two hundred nodes, leaving a network consisting of 61,640 variables and 3,441 nodes.

The reformulations discussed in this section can be quite helpful in reducing the size of the problem for solution purposes. However, from the decision maker's viewpoint, these reductions present some awkward problems. For example, given a solution to the first reformulation, the decision maker must decide how much each farm will ship in weeks four, five, and six, which although not difficult, can involve considerable handwork. Thus, recourse to these reduction techniques are desirable only if the

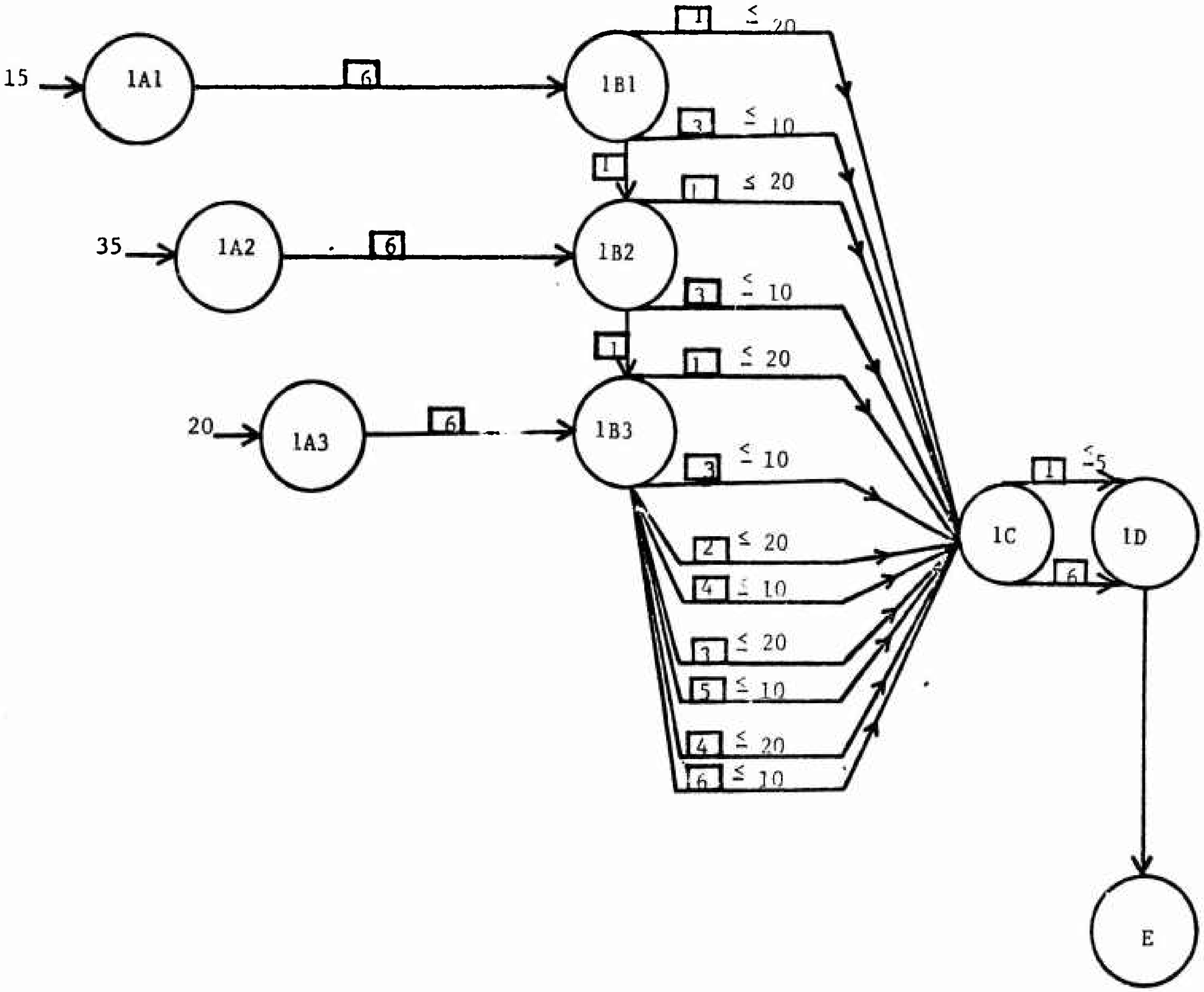

Figure 6

NETWORK AFTER FINAL REDUCTIONS

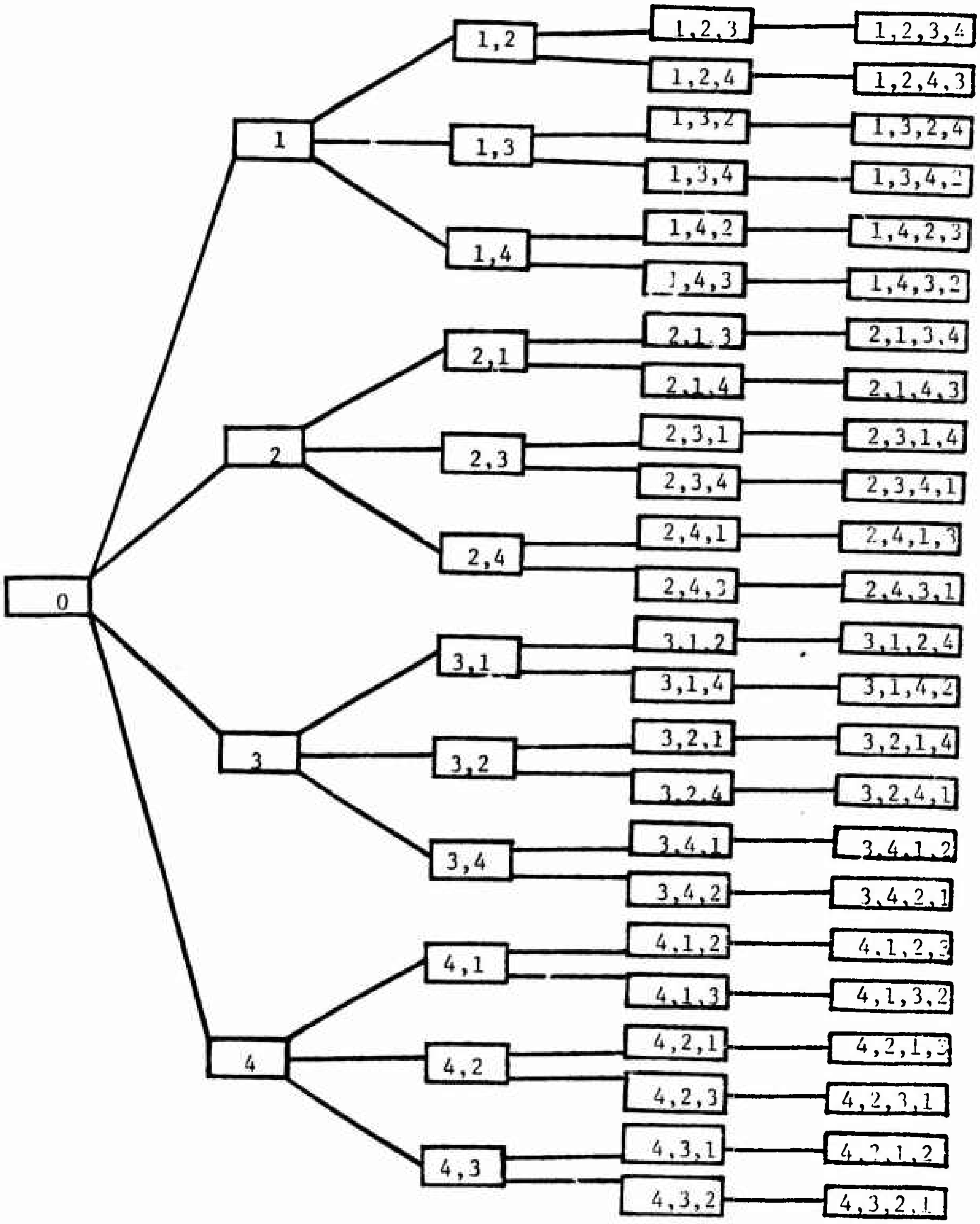

Figure 7

ORIGINAL DECISION TREE

larger model is not computationally solvable.

The final reduction considers only the weekly and seasonal variable costs. Let us now consider the effects of the seasonal fixed charges and obtain the final solution to the problem.

SOLUTION PROCESS

The resulting fixed charge minimum cost flow network problem can be solved by branch and bound procedures [2] using special purpose minimum cost flow network codes [1,3,4] to solve the associated subproblems. When these minimum cost flow network codes are used, the arcs from the level D nodes to the level E node will be given a cost of zero since these arcs have no costs which are proportional to the flow through these arcs. The subproblems to be tested in the branch and bound procedure consist of every possible "off" gin combinations. (Note: to turn off a gin in the network formulation simply corresponds to capacitating the arc from the level D node to the level E node associated with this gin to zero.) These combinations are illustrated in the decision tree given in Figure 7. Each node in this tree represents a combination of gins that have been turned off and not allowed to gin any cotton. For instance, to reach node |1,3|, one must remove gins 1 and 3 in that order. All gins not named in a node name are considered available for processing cotton.

Before describing the branch and bound procedure, observe that the topological structure of minimum cost flow network problems can be used to reduce greatly the size of the decision tree. In particular, a minimum cost flow network problem is feasible only if total supply is less than the total capacities of the gins available for processing. In our example the total production is 525 bales. The total capacity of all gins for the six

weeks is 930 bales. If gin 4 is removed, the capacity is reduced to 510 bales; thus, the problem is infeasible without gin 4 in operation. Also, simultaneously removing gins 1, 2, and 3 reduces capacity to 420 bales. Thus, without even attempting to solve any minimum cost flow problems, several combinations have been eliminated and need not be checked. The simplified decision tree is shown in Figure 8. This illustrates one computational advantage of the reformulation as a network problem. The primary computational advantage of the reformulation, however, is that each subproblem to be solved is a minimum cost flow network problem.

Consider starting at node $\boxed{0}$ of the decision tree in Figure 8. This assumes that the minimum cost flow network problem is to be solved with no gins turned off. Solving this problem yields a solution which uses all four gins and has an objective function value (including the fixed charge costs) of 7,005. Similarly, the testing of nodes $\boxed{1}$, $\boxed{2}$, and $\boxed{3}$ of the decision tree yield objective function values (including the fixed charge costs of 7,240, 6,895, and 6,755, respectively. These results indicate that gin 1 will always be used; therefore all nodes which involve turning off gin 1 can be eliminated. This leaves only node $\boxed{2,3}$ to be tested. Solving this problem yields an objective function value of 6,785. Consequently, the optimal solution corresponds to using gins 1, 2, and 4, leaving gin 3 idle and unused. The solution, which required less than 2 seconds on the CDC 6600 using SUPERK [1], is summarized in Tables III and IV.

While this is a small problem having only 6 nodes and 221 arcs, it does illustrate the speed with which such problems can be solved by exploiting the network structure of the model. Additionally, we believe

that the reduction of the real world cotton gin problem to a fixed charge minimum cost flow network problem with 3,641 nodes and 64,310 arcs and 20 "on-off" variables yields a problem which is computationally feasible to solve. In particular, we estimate that the state-of-the-art large scale minimum cost flow network code [4] can solve 100 subproblems per hour on a CDC 6600. Since the New Mexico Department of Agriculture will use this model during the next cotton production season, this estimate will soon be tested.

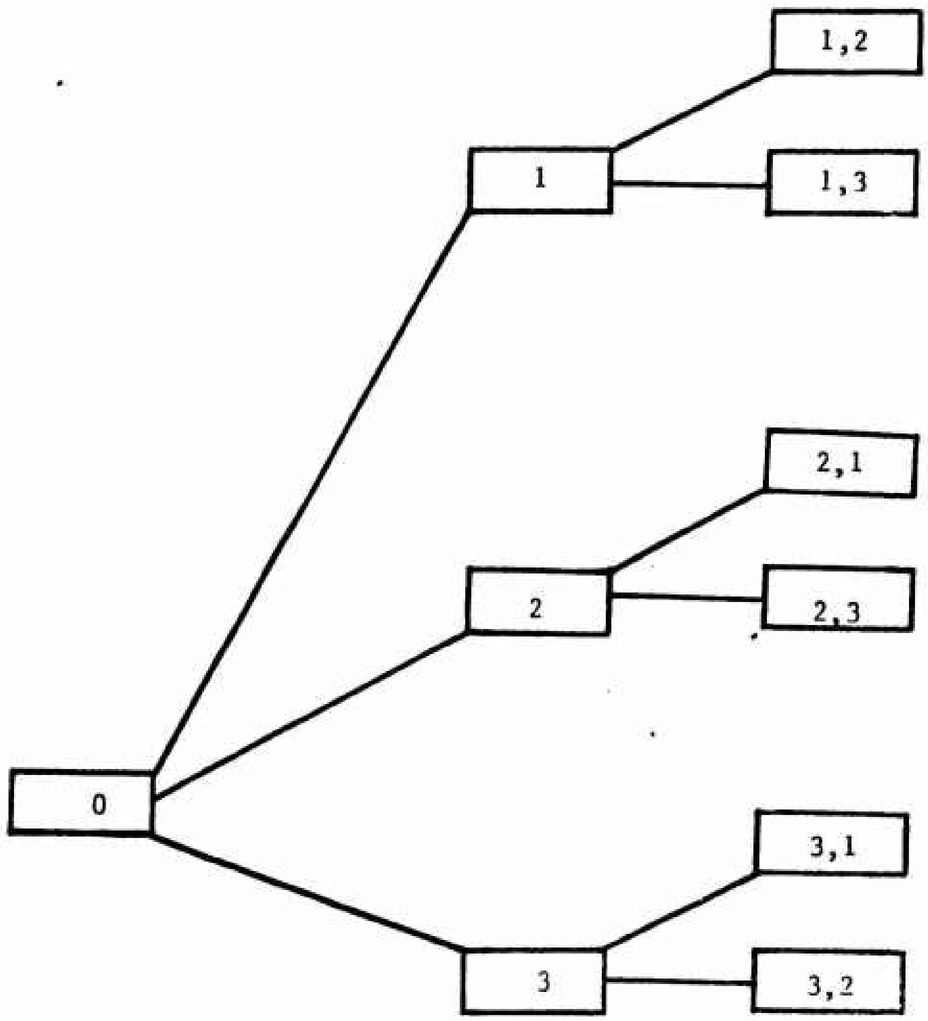

Figure 8

DECISION TREE AFTER REMOVAL OF INFEASIBLE CHOICES

TABLE III

FARM-TO-GIN SHIPPING ACTIVITY FOR OPTIMAL SOLUTION

Farm	Week	Bales Picked	Bales Shipped to Gin 1	2	4	Bales Stored
1	1	15		15		
	2	35		15		20
	3	20		15		25
	4	0	5	15		5
	5	0			5	
2	1	40			40	
	2	75	5		20	50
	3	60			60	50
	4	0	10		40	
3	1	30	30			
	2	45	25			20
	3	20	20			20
	4	0				20
	5	0	20			
4	1	15			15	
	2	50				50
	3	35	10			75
	4	0	10		20	45
	5	0			45	
5	1	15			15	
	2	50			50	
	3	20			10	10

TABLE IV

GINNING ACTIVITY FOR OPTIMAL SOLUTION

Gin	Week	Bales Shipped from Farm 1	2	3	4	5	Total Bales Shipped	Regular Shift Ginning	Overtime Shift Ginning
1	1			30			30	20	10
	2		5	25			30	20	10
	3			20	10		30	20	10
	4	5	10		10		25	20	5
	5			20			20	20	0
2	1	15					15	15	0
	2	15					15	15	0
	3	15					15	15	0
	4	15					15	15	0
	5	0					0	0	0
4	1		40		15	15	70	50	20
	2		20			50	70	50	20
	3		60			10	70	50	20
	4		40		20	10	70	50	20
	5		5		45		50	50	0

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